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Radial roller bearings with flexible rings: application to rotor dynamics and extension to multibody simulations

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Object:

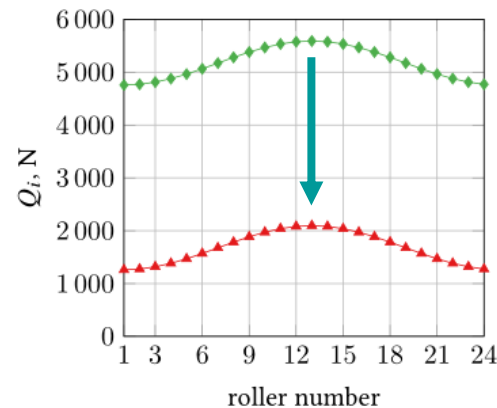
Radial roller bearings of modern gas-turbine engines with flexible thin-wall housings and hollow shafts.

Motivation:

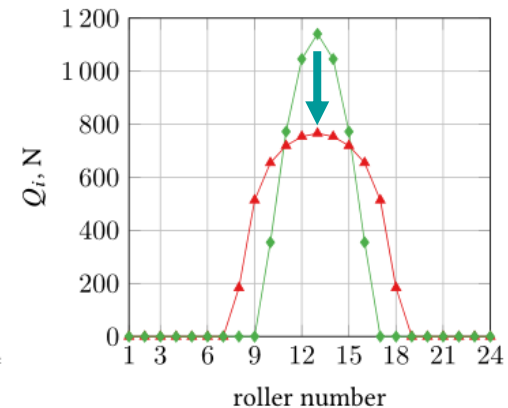
Development of approaches to account for bearing rings deformations in order to improve distribution of the internal radial forces acting on the rolling elements.



More accurate bearing life assessments.



negative clearance

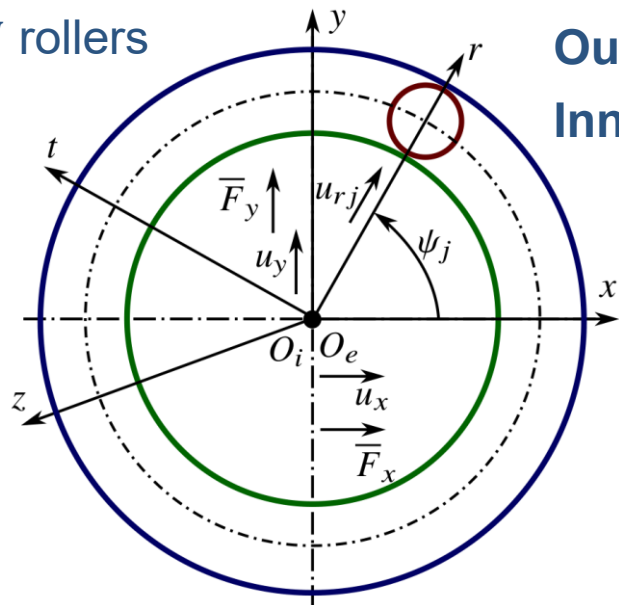


positive clearance

Bearing and roller equilibrium

N rollers

Outer ring – fixed
Inner ring – movable



Local contact:

$$Q_a = C_a L \delta_a^n$$

External force:

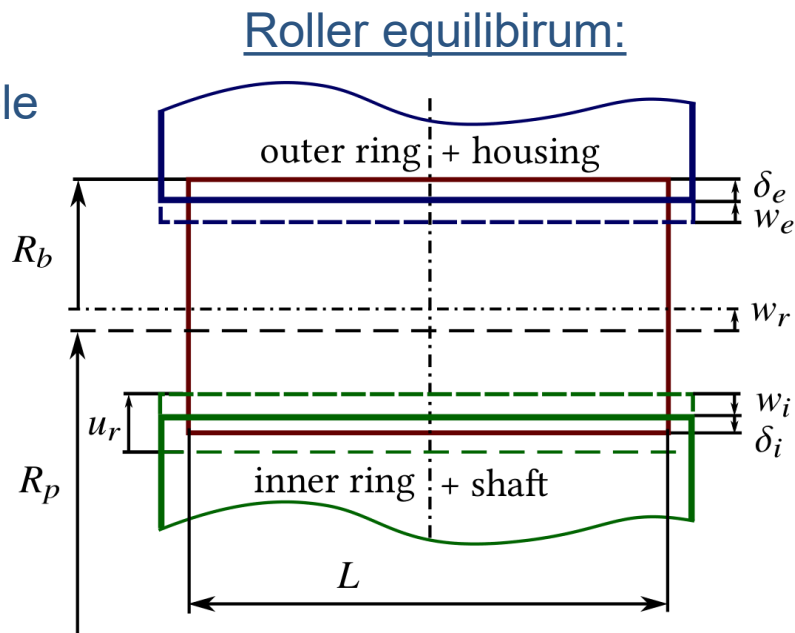
$$\bar{\mathbf{F}} = [\bar{F}_x \quad \bar{F}_y]^T$$

Inner ring displacements: $\mathbf{y} = [u_x \quad u_y]^T$

Inner ring equilibrium:

$$\bar{\mathbf{F}} + \sum_{j=1}^N Q_{ij} \mathbf{S}_j^T = \mathbf{0}$$

where $u_{rj} = \mathbf{S}_j \mathbf{y}$ and $\mathbf{S}_j = [\cos \psi_j \quad \sin \psi_j]$



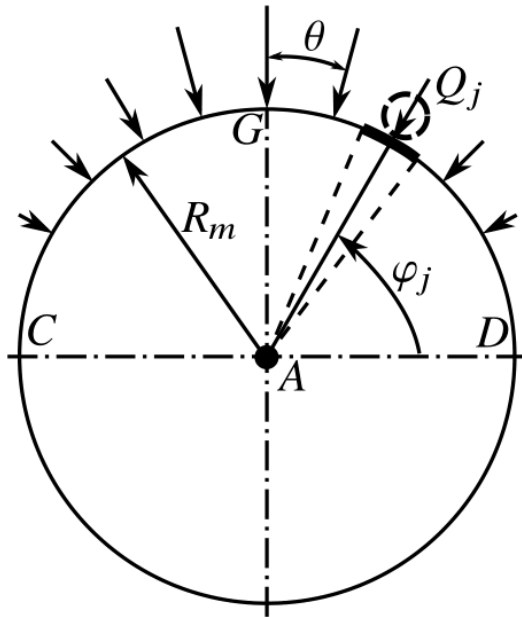
How to compute these quantities?

$$\frac{g}{2} + \delta_{ij} + \delta_{ej} + w_{ij} + w_{ej} - u_{rj} = 0$$

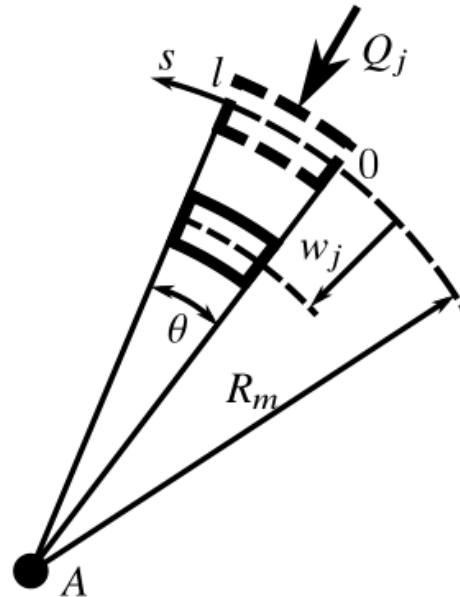
$N + 2$ equations

$N + 2$ unknowns: Q_{ij} and u_x, u_y

Option 1: ring as a set of independent segments



General deformation pattern



Simplified deformation pattern

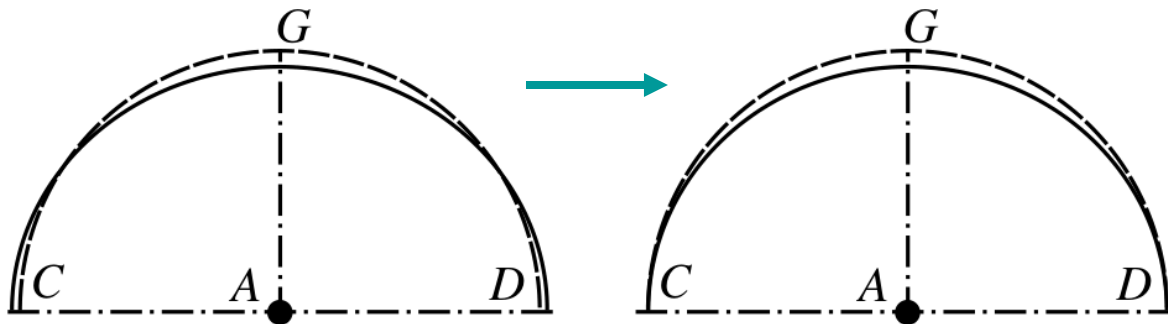
Segment strain:

$$\varepsilon = \frac{\Delta l}{l} = \frac{\theta(R_m - w) - \theta R_m}{\theta R_m} = -\frac{w}{R_m}$$

Tensile energy:

$$T = \int_0^{\theta R_m} \frac{1}{2} E \varepsilon^2 A ds = \frac{1}{2} E A \theta \frac{w^2}{R_m}$$

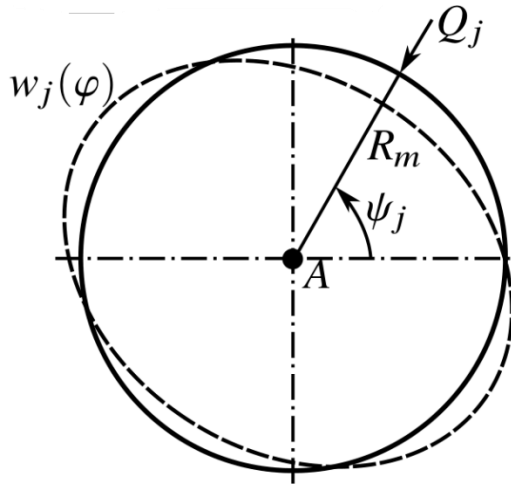
$$\rightarrow w_j = \frac{Q_j R_m}{E A \theta}$$



Notice:

Bending effects are neglected in this approach

Option 2: Fourier series expansion



Fourier series expansion of the ring deformed shape from an arbitrary j -th force:

$$w_j(\varphi) = Q_j \sum_{k=0}^{\infty} \lambda_k \cos(k\varphi) \quad \text{where}$$

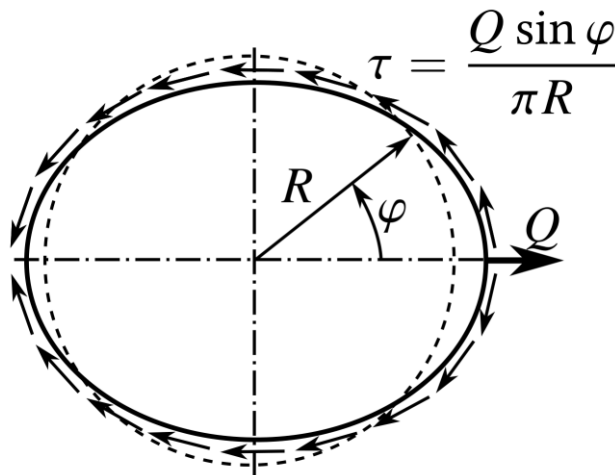
$$\lambda_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} w(\varphi) d\varphi$$

$$\lambda_k = \frac{1}{\pi} \int_{-\pi}^{\pi} w(\varphi) \cos(k\varphi) d\varphi$$

The resultant deformed shape:

$$w(\varphi) = \sum_{j=1}^N w_j(\varphi) = \sum_{j=1}^N Q_j \sum_{k=0}^{\infty} \lambda_k \cos k(\psi_j - \varphi)$$

How to compute coefficients λ_k ?



Analytical solution [Liu J. Y. and Chiu Y. P. (1974)]:

$$w(\varphi) = \frac{QR^3}{4\pi EI} \chi(\varphi) \quad \text{where}$$

$$\chi(\varphi) = \left((\varphi - \pi) \sin \varphi + \left(\pi\varphi - \frac{\varphi^2}{2} + \frac{3}{4} - \frac{\pi^3}{3} \right) \cos \varphi + 2 \right)$$

$$\lambda_k = \frac{R^3}{4\pi EI} \gamma_k$$

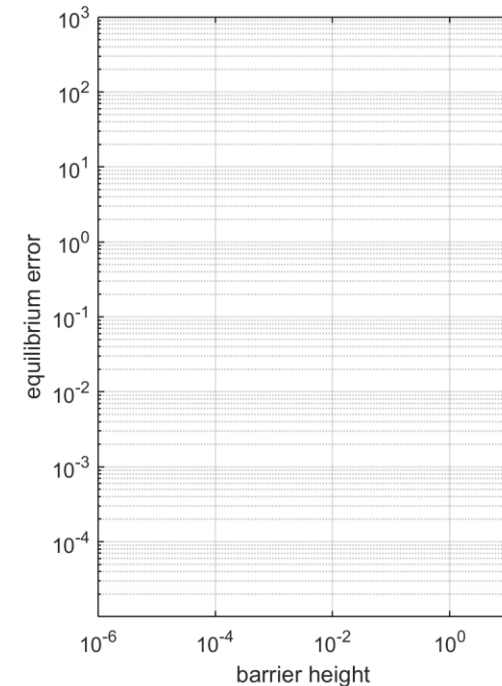
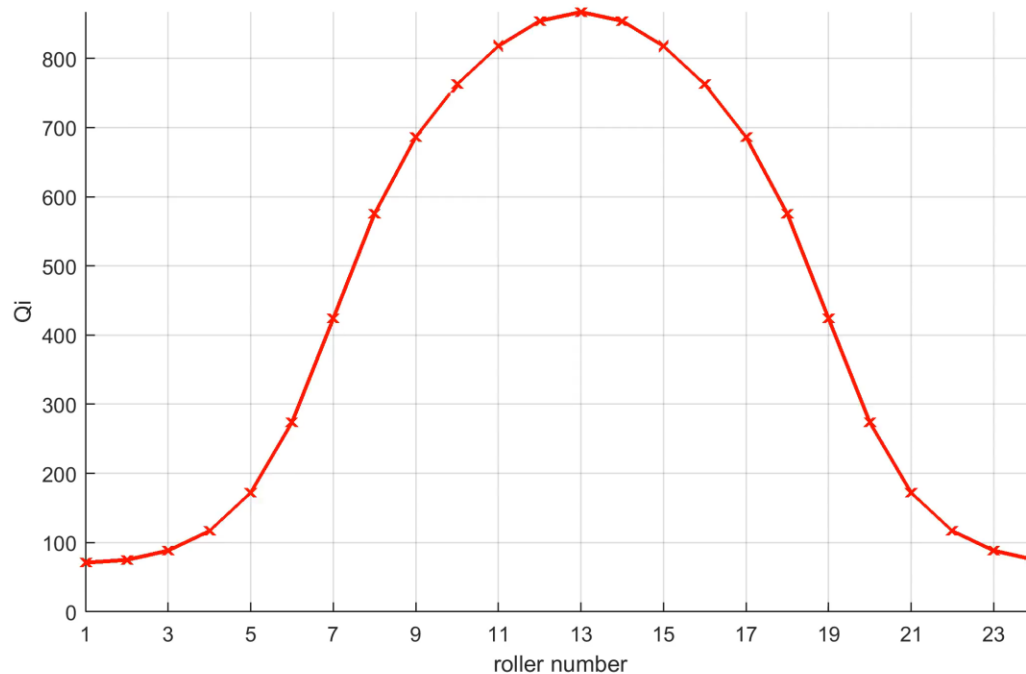
coeff	γ_0	γ_1	γ_2	γ_3	γ_4
value	0	0	$\frac{4}{9}$	$\frac{1}{16}$	$\frac{4}{225}$

Option 2: Fourier series expansion

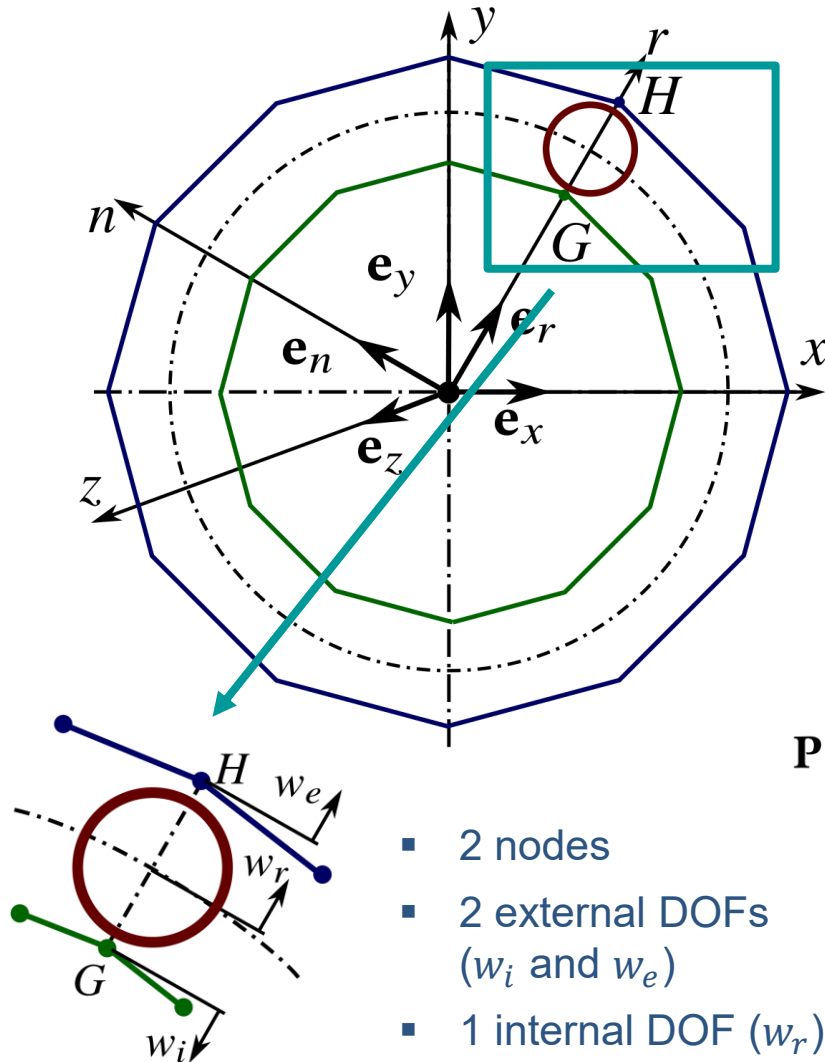
Limitations:

1. Does not account for tensile effects (crucial for negative clearance cases)
can be resolved using approach 1 as a preliminary analysis step

2. Problems when searching for nonlinear solution.



Option 3: simplified FE structural model



- 2 nodes
- 2 external DOFs (w_i and w_e)
- 1 internal DOF (w_r)

Rolling element flowchart:

1. Extract local DOFs values

$$w_i = -\mathbf{u}_G \cdot \mathbf{e}_r$$

$$w_e = \mathbf{u}_H \cdot \mathbf{e}_r$$

2. Solve nonlinear equation

$$Q_a = C_a L \delta_a^n$$

$$Q_e = Q_i + F_c \text{ with } \delta_i = -\frac{g}{4} - w_r - w_i$$

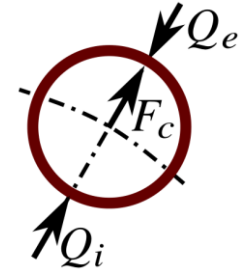
$$\delta_e = -\frac{g}{4} + w_r - w_e$$

3. Build local force vector and stiffness matrix

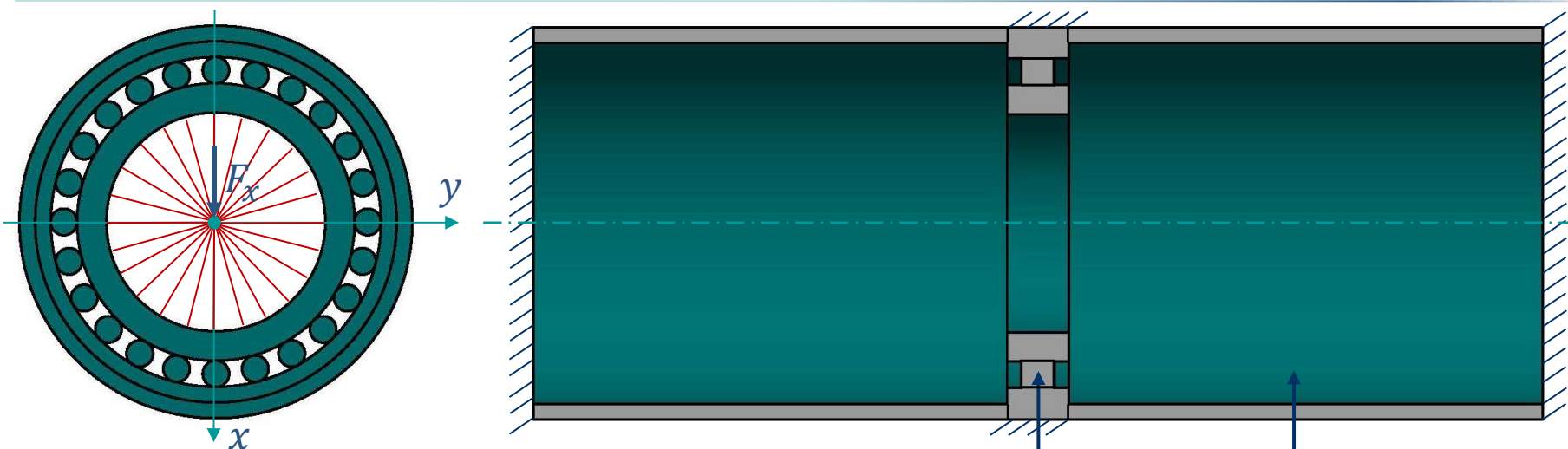
$$\mathbf{P} = \begin{bmatrix} Q_i \\ Q_e \end{bmatrix} \text{ and } \mathbf{K} = \begin{bmatrix} K_s & -K_s \\ -K_s & K_s \end{bmatrix} \text{ with } K_s = \frac{K_e K_i}{K_e + K_i}$$

4. Convert $\mathbf{P}$ and $\mathbf{K}$ to global coordinate system using

$$\mathbf{R} = \mathbf{e}_r \mathbf{e}_x^T + \mathbf{e}_t \mathbf{e}_y^T + \mathbf{e}_z \mathbf{e}_z^T$$



Numerical example



Bearing geometry:

RBE2 – for fixed rings case

$$N = 24$$

$R_p = 75\text{mm}$ – pitch diameter

$R_b = 6.5\text{mm}$ – roller radius

$R_s = 54\text{mm}$ – shaft inner radius

$R_h = 97\text{mm}$ – housing outer radius

$b = 30\text{mm}$ – rings width

$L = 16\text{mm}$ – roller length

Material:

$E = 2 \times 10^{11} \text{MPa}$ – elasticity modulus

$\mu = 0.3$ – Poisson ratio

$\rho = 7850 \text{kg/m}^3$ – density

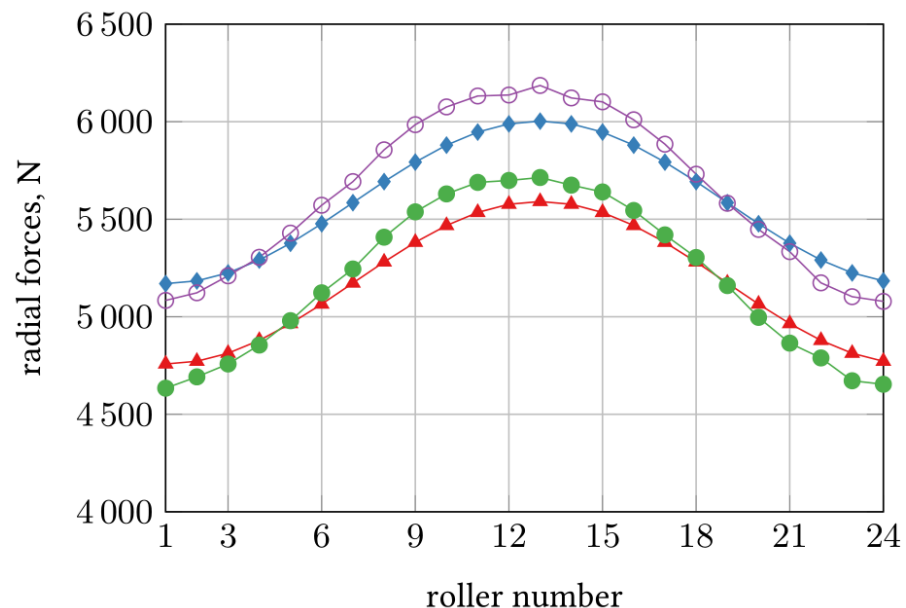
External loading:

$F_x = 5000 \text{N}$ – external force

$\omega = 12000 \text{rpm}$ – inner ring rotation speed

Numerical example: negative clearance

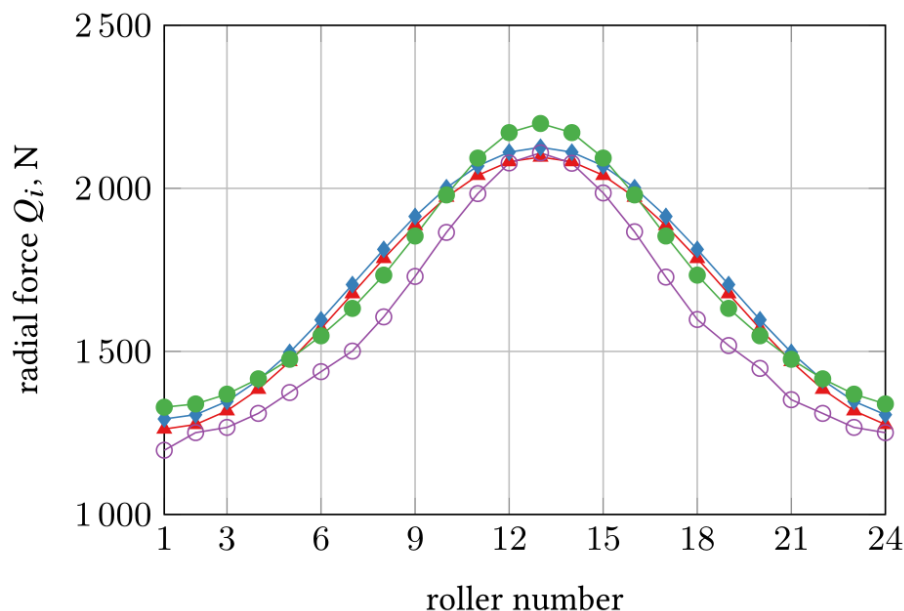
Negative clearance $g = -40\mu\text{m}$



—▲— Q_i —◆— Q_e independent segments approach
Fourier series approach

—●— Q_i —○— Q_e NX Nastran

rigid rings

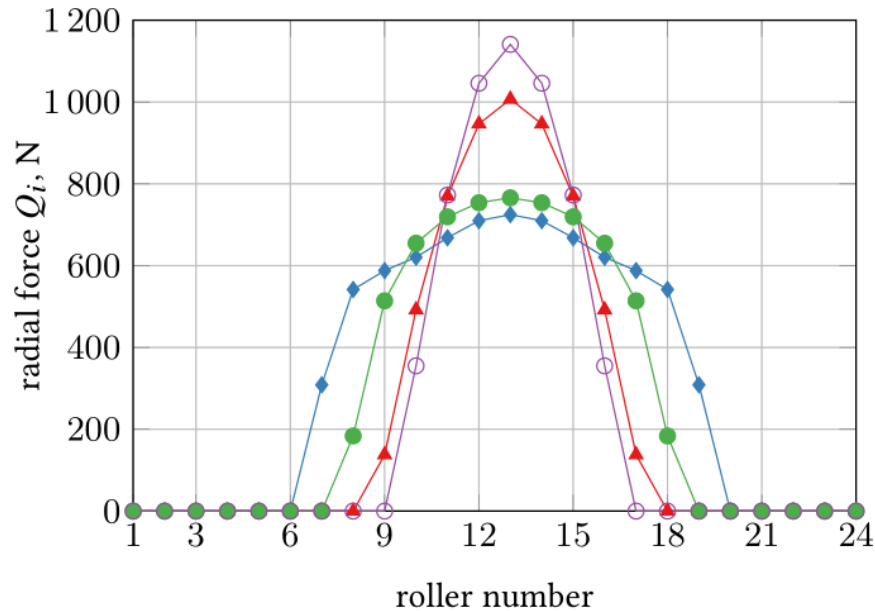


—▲— independent segments approach
—◆— Fourier series approach
—●— structural model approach (Dynamics R4)
—○— NX Nastran

flexible rings

Numerical example: positive clearance

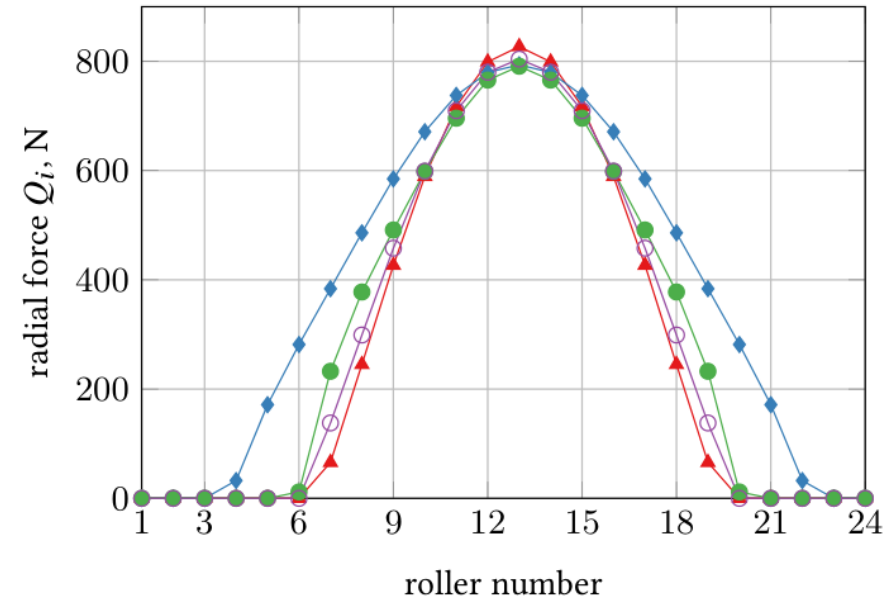
Positive clearance and nearly zero clearance cases



—▲— independent segments approach

—●— structural model approach (Dynamics R4)

$$g = 10 \mu\text{m}$$

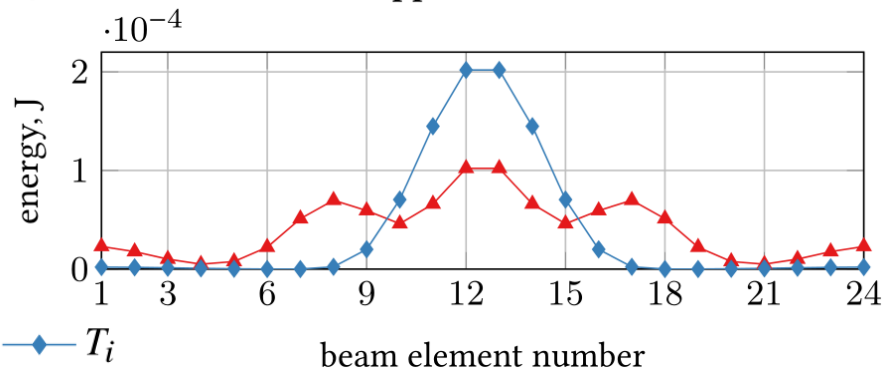
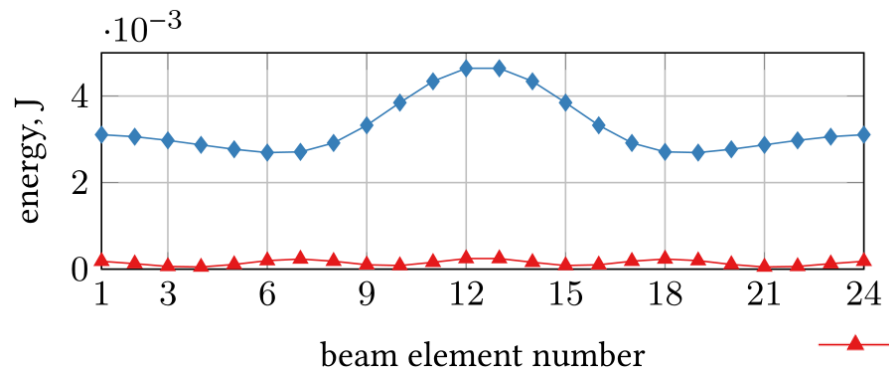
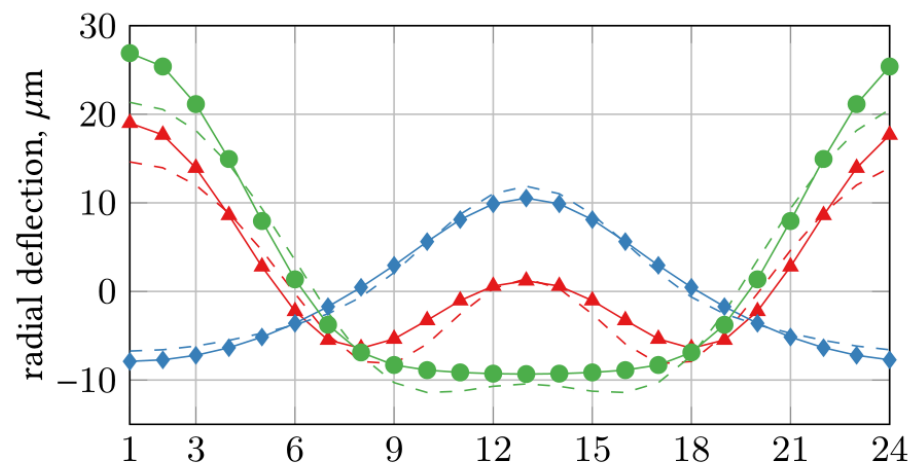
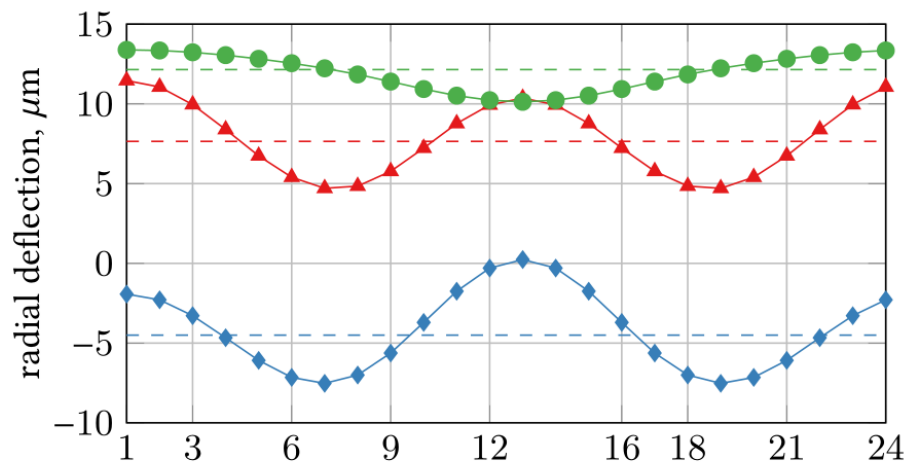


—◆— Fourier series approach

—○— rigid rings

$$g = -1.5 \mu\text{m}$$

Numerical example: bending vs tension



$g = -40\mu\text{m}$

$g = 10\mu\text{m}$

Extension to large displacements and rotations

Bearing finite element:

$$\bar{\mathbf{F}} + \mathbf{F}_E = \mathbf{0} \quad \text{-- bearing equilibrium}$$

Newton-Raphson procedure

$$\mathbf{K}_E \Delta \mathbf{y} = \bar{\mathbf{F}} + \mathbf{F}_E$$

$$\mathbf{F}_E = \begin{bmatrix} F_x \\ F_y \\ 0 \end{bmatrix} \quad \mathbf{K}_E = \begin{bmatrix} \frac{\partial F_x}{\partial u_x} & \frac{\partial F_x}{\partial u_y} & 0 \\ \frac{\partial F_y}{\partial u_x} & \frac{\partial F_y}{\partial u_y} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Some large rotation formulas:

θ – Euler rotation vector

$$\mathbf{Q} = \mathbf{I} + h_1(\theta) \mathbf{\Theta} + h_2(\theta) \mathbf{\Theta}^2 \quad \text{-- rotation tensor}$$

$$\mathbf{\Gamma} = \mathbf{I} + h_2(\theta) \mathbf{\Theta} + h_3(\theta) \mathbf{\Theta}^2 \quad \text{-- tangent tensor}$$

where

$$\mathbf{\Theta} = \text{skew}(\theta) \\ \theta = \|\theta\|$$

$$h_1(\theta) = \frac{\sin \theta}{\theta} \\ h_2(\theta) = \frac{1}{2} \left(\frac{\sin \theta/2}{\theta/2} \right)^2 \\ h_3(\theta) = \frac{1 - h_1(\theta)}{\theta^2}$$

Transformation to global coordinates system:

$$\mathbf{y}_G = \begin{bmatrix} \mathbf{u}_A \\ \theta_A \\ \mathbf{u}_B \\ \theta_B \end{bmatrix} \quad \mathbf{y} = \begin{bmatrix} u_x & u_y & u_z \end{bmatrix}^T = \mathbf{Q}_A^T \mathbf{u}_{BA} = \mathbf{Q}_A^T (\mathbf{u}_B - \mathbf{u}_A) \\ \delta \mathbf{y} = \delta \left(\mathbf{Q}_A^T \mathbf{u}_{BA} \right) = \mathbf{Y} \mathbf{\Gamma}_A^T \delta \theta_A + \mathbf{Q}_A^T \delta \mathbf{u}_B - \mathbf{Q}_A^T \delta \mathbf{u}_A$$

with $\mathbf{Y} = \text{skew}(\mathbf{y})$

$$\delta \mathbf{y} = \mathbf{H} \delta \mathbf{y}_G$$

$$\mathbf{H} = \begin{bmatrix} -\mathbf{Q}_A^T & \mathbf{Y} \mathbf{\Gamma}_A^T & \mathbf{Q}_A^T & \mathbf{0} \end{bmatrix}$$

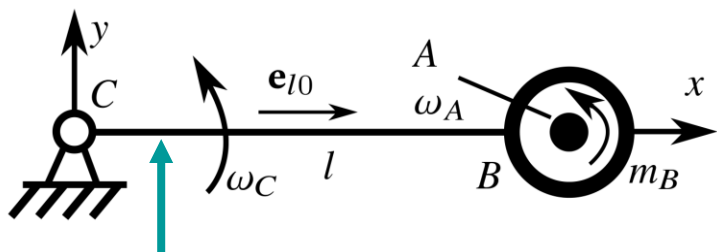
large spatial motion wrapper

$$\mathbf{F}_G = \mathbf{H}^T \mathbf{F}_E \\ \mathbf{K}_G = \mathbf{H}^T \mathbf{K}_E \mathbf{H}$$

Element vector and matrix in global coordinates

global state vector of a 2 nodes element

Extension to large displacements and rotations



Rigid link to impose constraints:

$$\mathbf{u}_{BC} = \mathbf{u}_B - \mathbf{u}_C - (\mathbf{Q}_C \mathbf{e}_{l0} - \mathbf{e}_{l0}) l = \mathbf{0}$$

$$\theta_{BC} = -\frac{1}{2} \text{axial} \left(\mathbf{Q}_B \mathbf{Q}_C^T \right) = \mathbf{0}$$

introduced via Lagrange multipliers

$$\Phi = \lambda_u \cdot \mathbf{u}_{BC} + \lambda_\theta \cdot \theta_{BC}$$

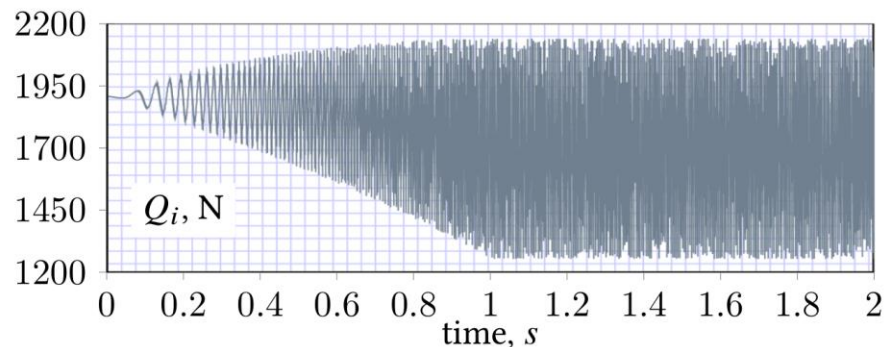
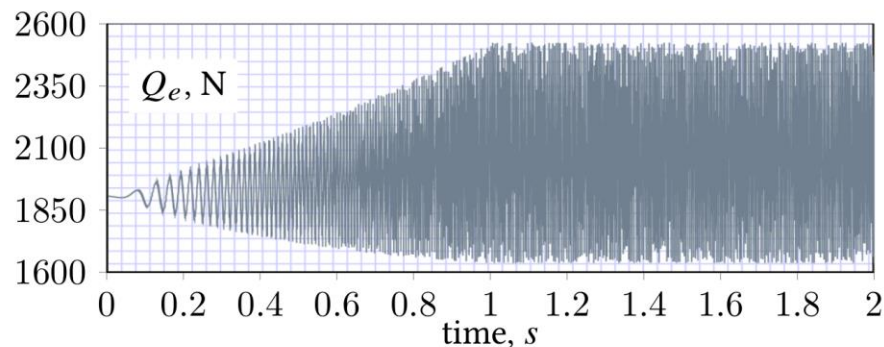
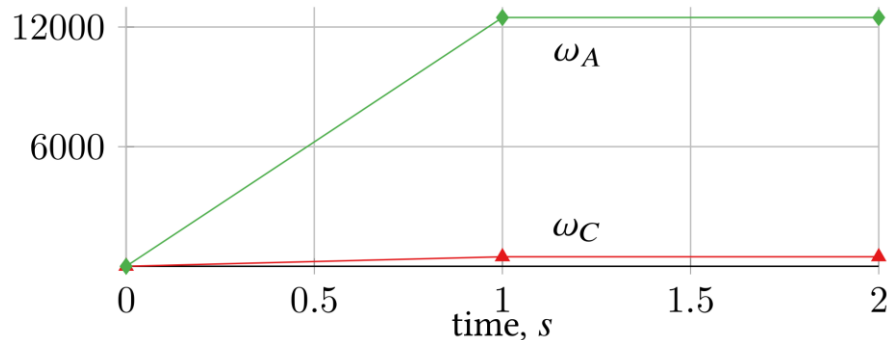
Rotation parameters:

$$m_B = 2\text{kg}$$

$$\omega_C = 50\text{rds} = 477\text{rpm}$$

$$\omega_A = 12000 + \omega_C = 12477\text{rpm}$$

$$F = m_B l \omega_C^2 = 5000\text{N}$$



Approaches comparison

Metrics	Independent segments	Fourier series approach	FE structural model
Accuracy for negative clearance cases	high	low / high (if fixed)	high
Accuracy for positive clearance cases	low	high	high
Performance	high	moderate	low
Robustness	high	moderate	high
Extensibility to multibody problems	easy	difficult	moderate
Profiled rollers	yes	no	yes
Easiness to build computational model	easy	moderate	complex

THANK YOU FOR YOUR ATTENTION!