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# TURBO-MACHINES WITH SLEEVE BEARINGS ROTORDYNAMICS ANALYSIS AT DESIGN STAGE

**Leontiev M.K. [1], Degtyarev S.A. [2], Kutakov M.N. [2]**

**[1] Moscow Aviation Institute (MAI)**
**[2] Alfa-Transit Co., Ltd**

## Summary

*The paper subject is the design problem of a rotor fitted with sleeve bearings. The main rotordynamics problems are bearing type selection, rotor stability limits and resonance regimes identification.*

## Introduction

Sleeve bearings are widely used in machinery especially in high power machines that operate at high loads such as steam turbines, centrifugal compressors, pumps, gas turbines, etc. The wide range of operating conditions requires multiple bearing types and design solutions that may be different even for the single bearing type. This is related to the rotor application, its size and mass, rotation speed, lubrication oil performance and supply method, bearing thermal state, etc.

The specific feature of sleeve bearings that differs them from the rolling ones is the circulation forces effect that may cause self-excited oscillations and the following rotor instability or shaft contact with the sleeve bearing surface. Thus, from the rotordynamics point of view the most important performance is the rotor operation below the stability border. The other important performances are the distance between the resonance and operation speeds and the dynamic loads upon bearings.

An important part of the sleeve bearing supported rotor design is the global dynamic system analysis. The essential part of the analysis is the calculation of the bearings static and dynamic performance.

The static parameters concerned to the shaft axis location under the external static loads are the following:

- Rotor specific eccentricity,
- Location and value of the film minimal clearance,
- Value of the film maximal specific pressure,
- Friction resulting power losses,
- Bearing oil leakages through the face surfaces and others.

In general, the sleeve bearings are non-linear elements of a rotor system, but their dynamic performance, stiffness and damping coefficients may be linearized within definite conditions, specifically for small shaft deflections from its steady state location.

The calculated bearing performance allow the following analysis:

- Rotor stability area, self-excited oscillations, their mode and frequency,
- Resonance response and magnitude of the vibrations caused by residual unbalances.

Analysis of the hydrodynamic bearing performance is well developed and widely presented, for example in standards ISO 7902-1-200 [1], API [2], book by V.M. Korovchinsky [3], handbooks [4, 5, 6] and numerous papers. On the other side the available codes for analysis of sleeve bearing supported rotor dynamics are surely not sufficient.

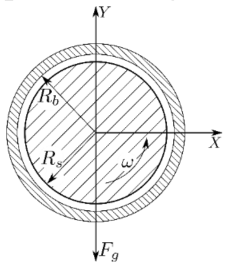
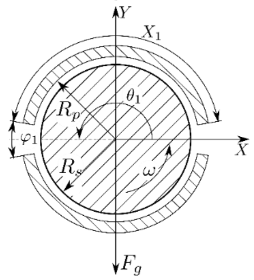
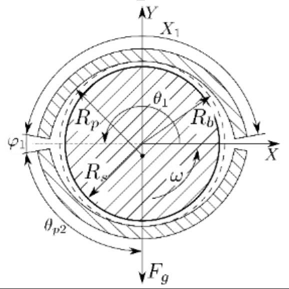
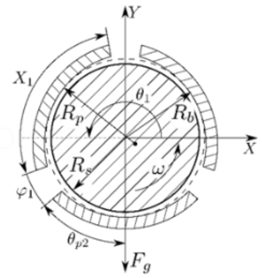
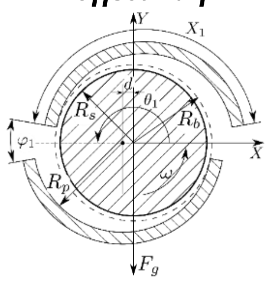
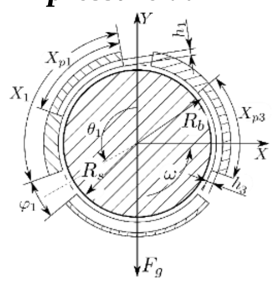
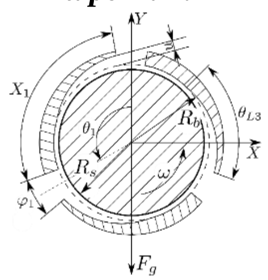
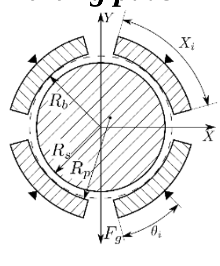
This paper briefly reviews the main sleeve bearing types, stages of rotor system analysis and selection of the sleeve bearing type. Also are shown capabilities of advances calculation codes, especially the DYNAMICS R4 [7] code. The demonstration example (ref. picture 1) is the BDAX 71-193ERH generator rotor by BRUSH company.

Picture 1. Generator rotor by BRUSH (www. brush. com)

**Bearing types**

Table 1 shows the most common sleeve bearing types [3, 5, 8]. The main dimensions in the table are the calculation input data.

*Table 1. Bearing types*

| ***plain bearing*** | ***axial groves*** | ***elliptic*** | ***multi-lobe*** |
|---|---|---|---|
|  |  |  |  |
| ***offset half*** | ***pressure dam*** | ***taper land*** | ***tilting pads*** |
|  |  |  |  |

**Plain bearing**

The simplest fixed shape bearing type has a cylinder contact surface. In the shaft central position the clearance is constant along the contact surface length and forms one hydrodynamic wedge that produces a high dynamic load capacity. On the other side the large non-uniformity of the liquid layer pressure in the clearance results in large coupled cross-influence of the hydraulic forces components in the liquor film. Among the sleeve bearings this type has the lowest stability border that may be improved by the clearance reduction, but this reduces the oil flow, increases the energy losses and temperatures which causes the bearing wear. So, this bearing type is mostly used for heavy rotors with low rotation speeds, the type advantages are its simple design and low price.

**Axial groove bearing**

This type bearing has dynamic performance similar to the plain bearing's ones. The cylindrical contact surfaces are split by 2 to 4 axial grooves. The stability border for 2 axial grooves with one hydrodynamic wedge is low. It may be increased together with the load capacity by reduction of the clearance but it causes a remarkable increase of the bearing operation temperature. An increase of the groove's numbers up to 4 with a turn towards the external load, for example for 45 degrees increases the wedges number to 2 and remarkably moves the stability border from the operation regime. This bearing design and manufacturing are simple.

This type bearing is applied to heavy machines with low rotation speeds such as gas turbines, power generators, compressors and pumps, gearboxes.

**Elliptic bearing**

This bearing is sometimes called "lemon-shaped". It is similar to the previous type but the contact surfaces clearance is reduced in the vertical axis direction. This clearance change along the surface is called "preload" or "shape coefficient" and is described as the ratio of distance between the segment curvature center and the bearing geometrical center to the radial clearance $\mathrm{mf} = 1 - \mathrm{C_b}/\mathrm{C_p}$. This change of the bearing shape remarkably re-distributes the clearance forces and increases the stability zone. The mf value is taken from analysis and the main bearing manufacturers usually keep it in the range of 0.5 to 0.6. It is worth mentioning that in the elliptic bearing is retained a considerable cross-influence of the forces in two perpendicular planes which does not allow a complete elimination of self-excited oscillations. This bearing has good damping at operating speeds.

Manufacturing of this type bearing is relatively cheap and simple. Maybe it is the most popular type for the low and middle speed rotors. On the other side the change of a cylindrical bearing to this type is accompanied by larger oil flow through the side slots, higher pressure on the bearing surfaces and some increase of the lubrication oil temperature. Besides this this bearing has orthotropic stiffness and damping so the bearing analysis must involve the static load direction.

**Multi-lobe bearing**

This bearing contact surface consists of a few parts, usually three or four. Like in the elliptic bearing all contact surfaces have varying clearances, or "preload". The stability border is higher than in the elliptic bearing. The load capacity and rotation speed are high, the friction power losses are high. The contact surfaces shapes are determined by the offset, or angular displacement of the contact surface curvature center against the bearing center. The contact surfaces shapes may symmetric with offset=0.5, or asymmetric with offset>0.5 so in terms of the bearing rotation direction it may be twin-directed, e. i. with two rotation directions, or one-direction.

Usually this type is used in small high-speed machines with high stability limits. Its manufacturing requires high accuracy equipment. The oil leakages through face surfaces is smaller than in the elliptic bearing.

**Offset half bearing**

This type continues the axial groove bearing concept. Its upper and lower contact surfaces are sideways displaced in opposite directions. This displacement produces a non-symmetric clearance; smaller in vertical direction that improves the stability border but is not applied to changes of the shaft rotation direction.

This bearing manufacturing is simple and cheap. It is used for heavy rotors.

**Pressure dam bearing**

This bearing geometry is also fixed. Its classic configuration has 2 cylindrical contact surfaces. The upper unloaded part has a groove up to 3 clearances deep, or a dam. Vertical rotor machines may have configurations with 3 or more operating surfaces with dams in each of them. A dam forms a step change of the segment surface. Inertia effects in oil may produce sharp peaks of high pressure that cause the shaft motion to larger eccentricities. These large eccentricities increase stability because the support stiffness becomes more asymmetric and the oil layer damping also grows but the power losses also grow because of the large loads. These bearings are not convenient in the supports with varying load direction or changing direction of rotation but the load amount may verify in a broad range.

This type of bearing may be used in steam turbines and in high speed machines, or gearboxes for improvements of the stability range better than other fixed configuration bearing types. This bearing manufacturing is requiring more labor because the bearing includes separate pads that need additional cutting operation. Also, these bearing needs high manufacturing accuracy and high oil purity due to the possible dirt and wear particles accumulation in dams.

**Taper land bearing**

This type continues the concept of offset half bearing. It has three contact surfaces with the clearances narrowing along the length, or the minimal clearance is offset to the surface edge. This design improves stability and has good damping and load capacity but it is not used for the changing rotation direction. Its manufacturing is complicated. It is applied to turbo-chargers.

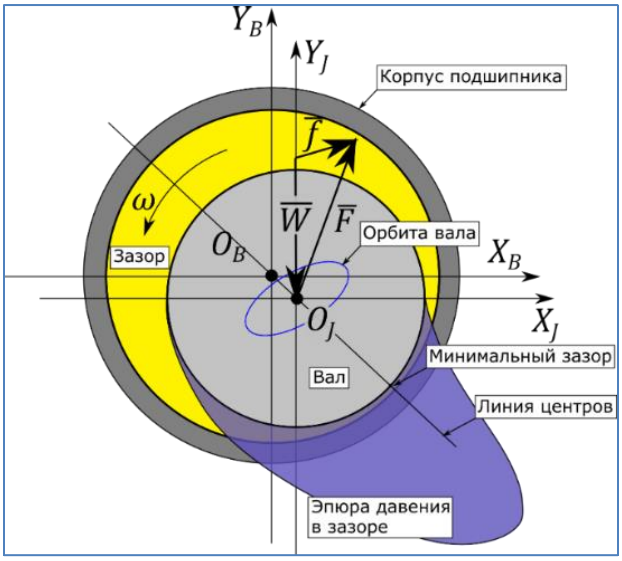
**Tilting pads bearing**

The tilting pads bearing is the only sleeve bearing type that is practically not subject to the stability loss effect. Usually it has 4 or 5 segments, or pads with various suspensions. Each pad may turn around its support as to meet the distribution of hydrodynamic wedge forces. It results in the de-stabilizing tangential forces reduced to almost zero so the bearing is not the source of the system unstable behavior, or self-excited oscillations. The lubrication oil flow may be smaller than in the elliptic bearing. The bearing is used in high speed rotors including the supercritical ones.

The bearing has smaller load capacity than the fixed geometry ones, larger energy losses and poor resonance damping. The design is complicated, its manufacturing is labor consuming and expensive.

**Theory assumptions**

The hydrodynamic sleeve bearing theory assumes that at normal operation conditions the rotor makes precession motions around a stationary position and the oscillation magnitude is below half of the operating clearance. Then the bearing reaction force $\vec{F}$ may be split into two parts, the static part $\vec{W}$, for example weight, and the dynamic part $\vec{f}$ due to the shaft axis deviation in its precession in the oil filled clearance caused by the harmonic violating force, in other words $\vec{F}=\vec{W}+\vec{f}$ (picture 2). The static equilibrium shaft position in the bearing may be determined as $\vec{F}+\vec{W}=0$.




Picture 2. Force vectors and the rotor motion orbit against its equilibrium position, $O_B$ – bearing geometrical axis, $O_J$ - static position of the shaft center

The vertical and horizontal components of the force on the shaft neck may be determined as an integral of the static pressure field $p(\varphi, z)$ that may be obtained from a numerical solution of the 2-D Reynolds equation [3].

$$f_x = -R\int_0^{2\pi}\int_0^{L} p(\varphi, z)\ \cos(\varphi)\ d\varphi dz,$$
$$f_y = -R\int_0^{2\pi}\int_0^{L} p(\varphi, z)\ \sin(\varphi)\ d\varphi dz,$$

$L$ – bearing width,
$R$ – shaft neck radius,
$\varphi$ – angular coordinate in annular direction,
$z$ – axial direction coordinate.

The forces gradients or the instant values of dynamic stiffness coefficients $k$ and the damping coefficients $c$ are determined as the following:

$$k_{xx} = -\frac{\partial f_x}{\partial x};\ k_{xy} = -\frac{\partial f_x}{\partial y};\ k_{yx} = -\frac{\partial f_y}{\partial x}; k_{yy} = -\frac{\partial f_y}{\partial y},$$
$$c_{xx} = -\frac{\partial f_x}{\partial \dot{x}}; c_{xy} = -\frac{\partial f_x}{\partial \dot{y}}; c_{yx} = -\frac{\partial f_y}{\partial \dot{x}}; c_{yy} = -\frac{\partial f_y}{\partial \dot{y}}.$$

The force $\vec{f}$ matrix form is:

$$\begin{Bmatrix} f_x \\ f_y \end{Bmatrix} = -\begin{bmatrix} k_{xx} & k_{xy} \\ k_{yx} & k_{yy} \end{bmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix} - \begin{bmatrix} c_{xx} & c_{xy} \\ c_{yx} & c_{yy} \end{bmatrix}\begin{Bmatrix} \dot{x} \\ \dot{y} \end{Bmatrix}.$$

The forces substitution into the motion equations forms equation of the rotor system free oscillation:

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{0\},$$

$[M]$– inertia matrix;
$[C]$ – damping matrix;
$[K]$ – stiffness matrix.

This equation solution is $x = Xe^{-pt}$. The calculation of natural frequencies $p_i$ and oscillation modes $X_i$ is reduced to the eigenvalues and modes problem.

The damped system complex frequency $p$ consists of two summands and may be obtained in the DYNAMICS R4 as

$$p = (\omega_d \pm i\lambda),$$

$i$ – imaginary unit;
$\omega_d$ – damped frequency;
$\lambda$ – damping factor.

The specific damping coefficient may be obtained as

$$\zeta = \frac{\lambda}{\sqrt{\lambda^2 + \omega_d^2}}.$$

The negative value of coefficient $\zeta$ in the damped frequency $\omega_d$ shows that the rotor becomes unstable with the oscillation mode that corresponds to this frequency and the oscillation magnitude grows.

**Sleeve bearing analysis**

The DYNAMICS R4 code has the hydrodynamic sleeve bearing analysis module DynFB where the performance calculation is based on a numerical solution of 2-D Reynolds equation. The equation is solved by the finite differences method with the user determined boundary conditions, for example number and dimensions of axial grooves, dams and grooves in segments that change the contact surfaces geometry, etc.

The algorithm employs the following assumptions added to the basic theoretical ones:

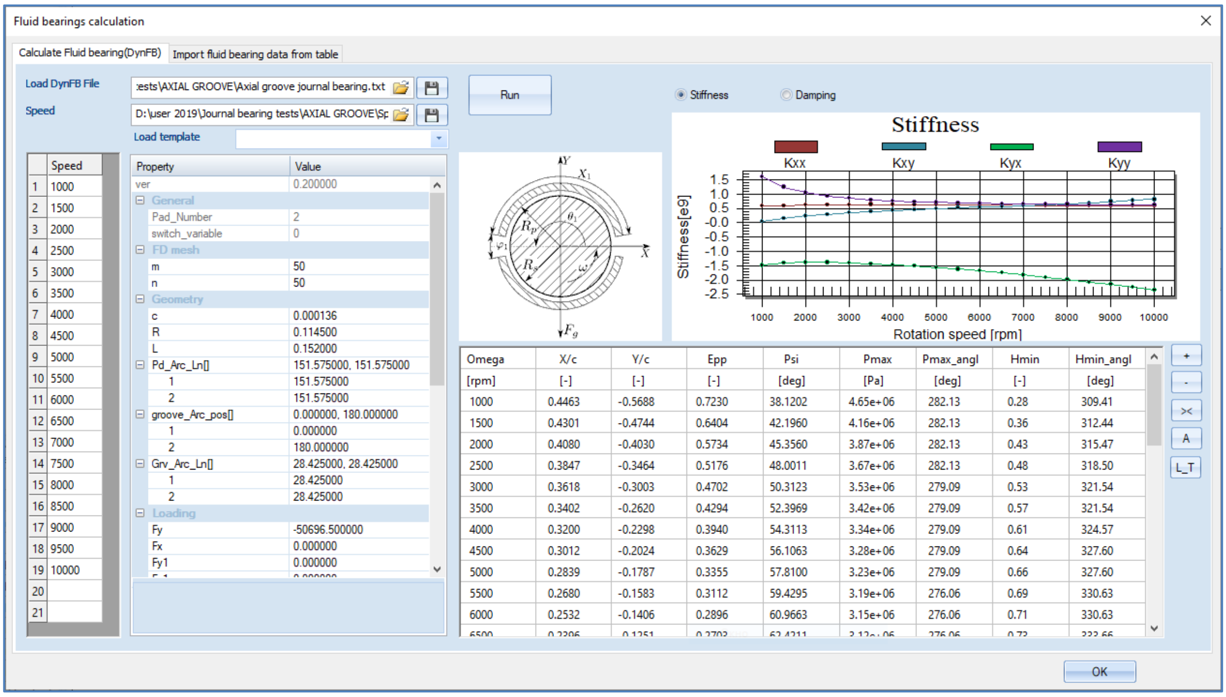
- The bearing oil is of the Newton type which means the liquor layer shear stresses are proportional to the velocity gradient,
- Shaft and housing surfaces are absolutely hard and isothermal,
- Heat transportation between the neighboring segments is involved by the heat transportation coefficient,
- Shaft misalignment in the bearing is not considered.

The main input data for the bearing static and dynamic performance analysis are the following:

- Supporting segments dimensions,
- Bearing clearance,
- Bearing length and shaft diameter,
- Static load upon the support,
- Oil performance, viscosity, specific weight, thermal expansion coefficient,
- Rotor rotation speed,
- Oil supply pressure.

Picture 3 shows the DynFB window for calculation of the bearing performance. One can select a bearing type including with addition of a new configuration.

The bearing analysis determines various parameters at each rotation speed including static performancce, stiffness and damping coefficients that determine the rotor dynamics. The stiffness and damping coefficients array is automatically transmitted to the preliminary prepared rotor model which forms the quazy-linear rotor model. After this the rotor model with sleeve bearings is completely ready for the dynamic performance analysis.




Pic. 3. The DynFB window for calculation static and dynamic bearing performance

### Creation and analysis of the generator model

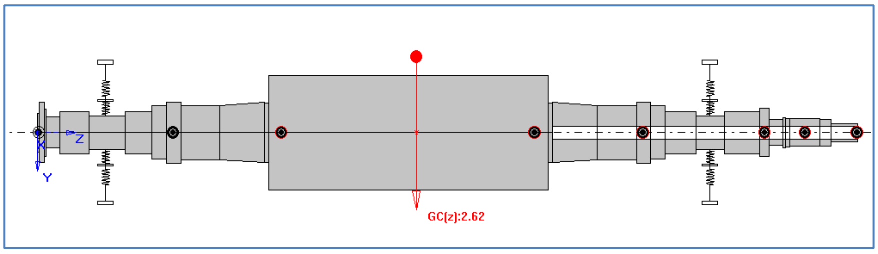
This chapter subsequently discloses the analysis steps for the generator rotor dynamic system in the DYNAMICS R4 code. All tittles of graphs and output parameters are given in the code definitions.

***Step 1.*** Design of the generator rotor dynamic system model based on its dimensions and mass-inertia parameters.

Generator parameters [9]:

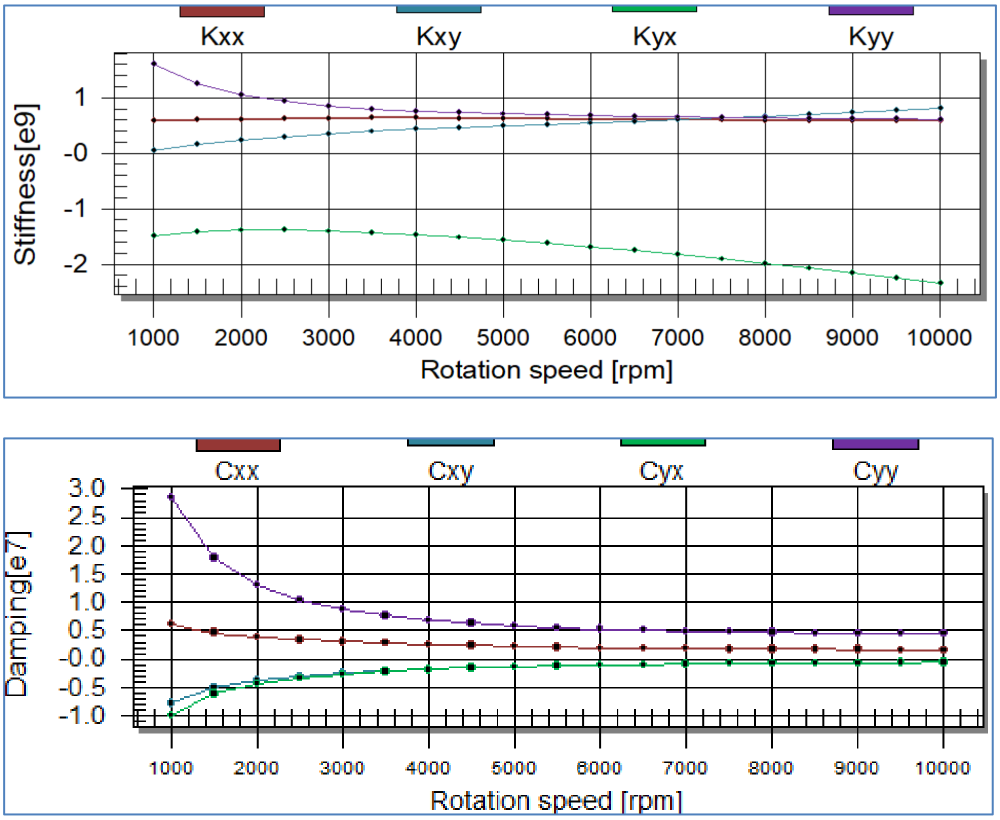
- Generator motor mass - 10636 kg;
- Bearing effective length - 152 mm;
- Clearance - 0.136 mm.
- Radius - 114.5 mm;
- Lubrication oil - ISO VG 32;
- Oil dynamic viscosity at 70$^{O}$C - 0.01436 N sec/m$^2$;
- Operating speeds – 50 or 60 Hz;
- Unbalance – 10000 gmm.

The designed model is shown in picture 4.




Pic. 4. Generator rotor model designed in DYNAMICS R4

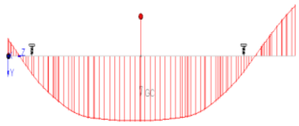
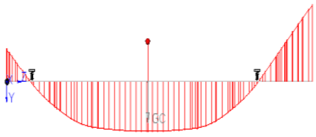
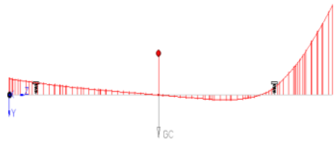
***Step 2.*** The bearing type is selected, its static and dynamic performance are calculated. Picture 5 shows example coefficients for a cylindric bearing with 2 axial grooves. The coefficients are sent to the support  modules of the rotor model. The bearing parameters and the weight reation forces are equal.




Pic. 5. Stiffness coefficients [N/m] and damping [Ns/m] of a cylindric bearing with 2 axial oil supply grooves

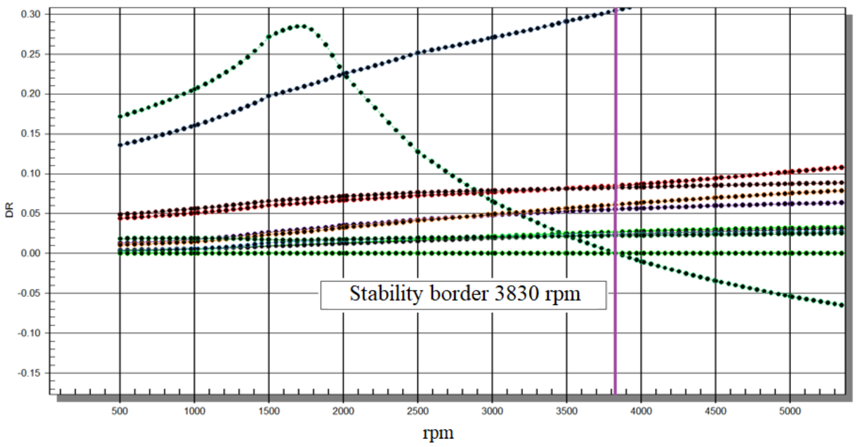
***Step 3.*** Calculation of the rotor critical rotation speeds and oscillation modes (table 2). The table shows the generator rotor flexible and supercritical.

Table 2. Critical speeds and oscillation modes

| 1963 rpm | 2625 rpm | 6056 rpm |
|---|---|---|
|  |  |  |

It is known that the rotors with cylindrical sleeve bearings have the stability border near or above the twice first critical speed [4], or in this example about 3926 rpm.

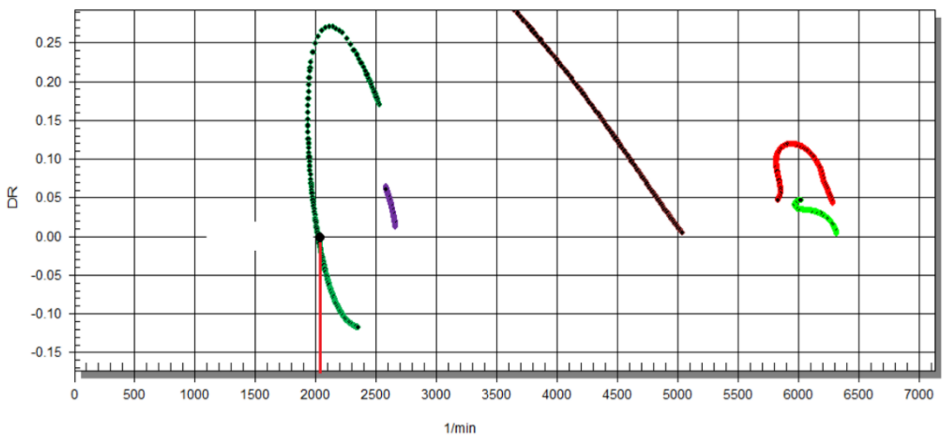
***Step 4.*** The stability limit may be specified by calculation of the stability map, or dependence of the specific damping coefficient upon the rotor speed (pic. 6). The zero value of the specific damping coefficient determines the rotor speed that corresponds to the auto-oscillation occurrence, or the rotor stability loss. In this case this speed is equal to 3830 rpm.




Pic. 6. Specific damping coefficient vs rotation speed

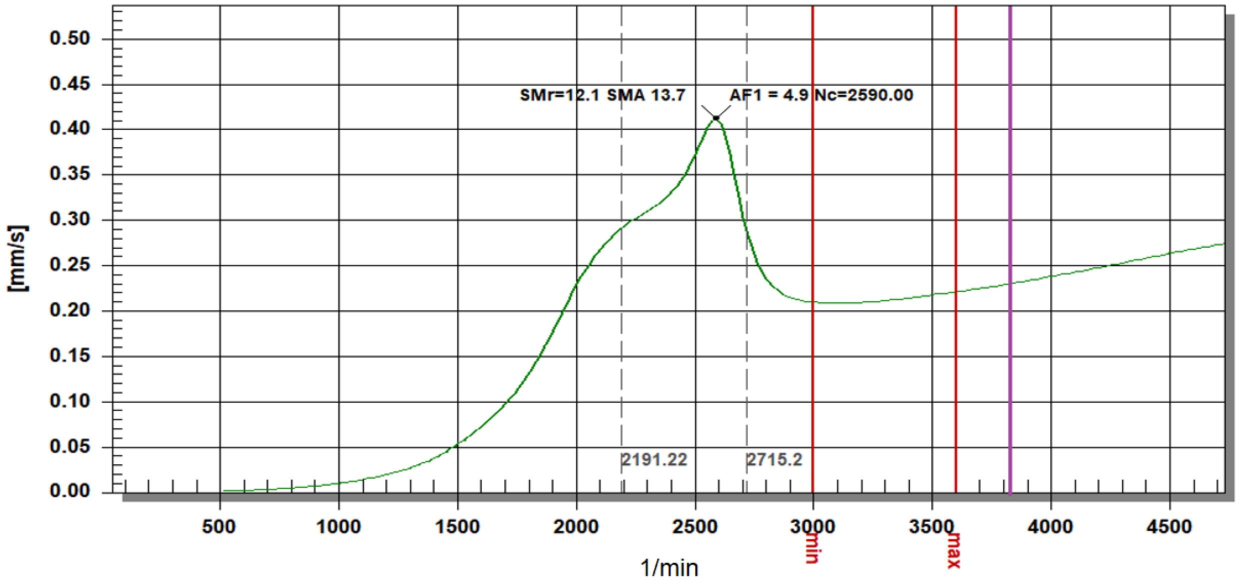
***Step 5.*** The stability loss oscillation ferequency and mode are determined by the relations of reduced damping coefficient and the natural oscillation frequency (pic. 7). The reduced damping coefficient zero value determines the natural frequency and mode that correspond to the stability loss. The calculated auto-oscillation frequency corresponds to the natural mode of 2004 1/min frequency.

The obtained results show a small distance between the stability border and the generator operation speed. The margins to 3000 rpm and 3600 rpm operation speeds are 830 rpm and 230 rpm respectively. This requires a bearing type change, for example to the “lemon” type.




Pic. 7. Dependence of relative damping coefficient upon natural frequency

***Step 6.*** Other factors to be considered at the design stage are distances from the operation speed to the critical speed, resonance vibration level and amplifying coefficient. The API RP 684 standard [2] describes the calculation method for the nominal operation regime location in relation to resonance regimes. Picture 8 shows the calculated magnitude-frequency generator performance in the rotor support.




Pic. 8. Generator magnitude-frequency performance.
Nc – rotor critical speed, AF1 – magnifying coefficient, SMr – necessary margin, SMa – actual margin

Vertical lines show the operating speeds 3000 rpm and 3600 rpm and the stability limit 3830 rpm. The plots correspond to the linearized dynamic system model. The sleeve bearings are linearized with the assumption of small oscillation motions of the rotor around its steady position that is determined by the external static loads. After the stability loss the rotor will oscillate with the magnitudes comparable with the bearing clearance which is beyond applicability of the linearized bearing model, so the calculation results may be applied up to the stability limit.

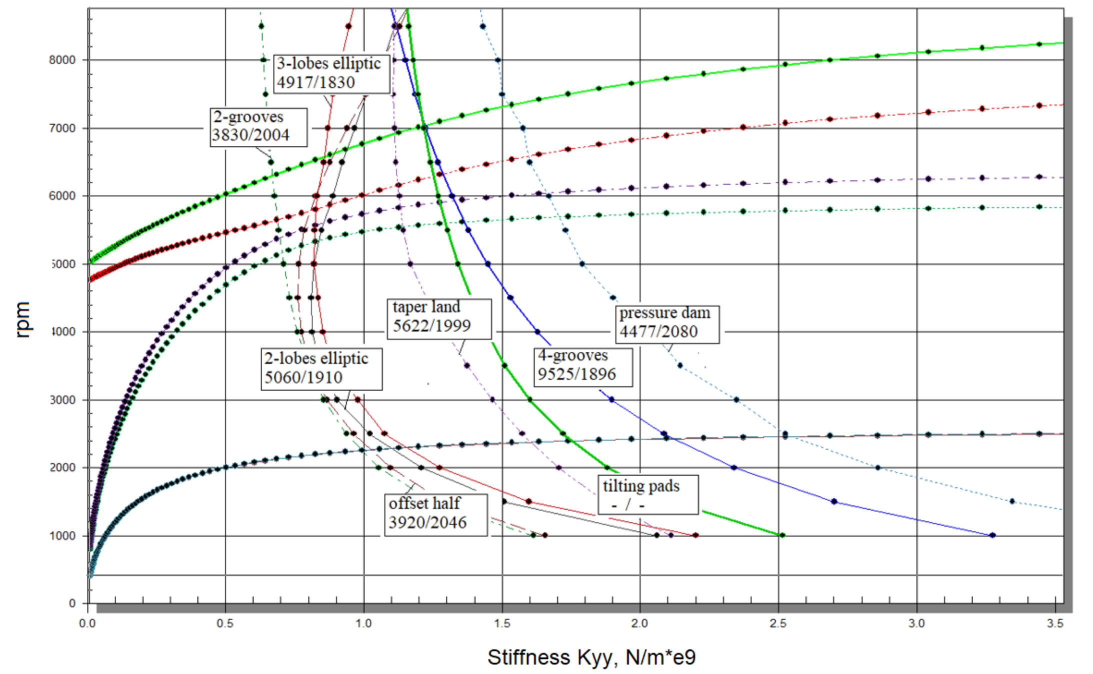
**Different bearings analysis**

The low stability limit of the cylindric bearing forces a chenge of bearing type. The other type should have smaller cross-coupling components of the film hydraulic force which will provide a higher calculated stability border.

Besides the stability limit there are other important parameters and specifically the critical speed location and the resonance tuning margin.

Critical speeds with different sleeve beraring types for the first approach may be compared with the critical speeds map obtained from the model parametric study. Usually the map is used for isothropic systems with one variable coefficient. Sleeve bearing stiffness and damping are described with matrixes 2x2, and the critical speeds map versus the vertical stiffness coefficient Kyy gives an approximate presentation of the actual critical speeds.

Picture 9 shows the critical speeds map and the stiffness coefficients Kyy related to the rotation speed for different bearing types. In the map the bearing type name is followed with two values, the stability limit of the rotor system with this type of bearing and the natural frequency of the mode that correspond the stability loss.



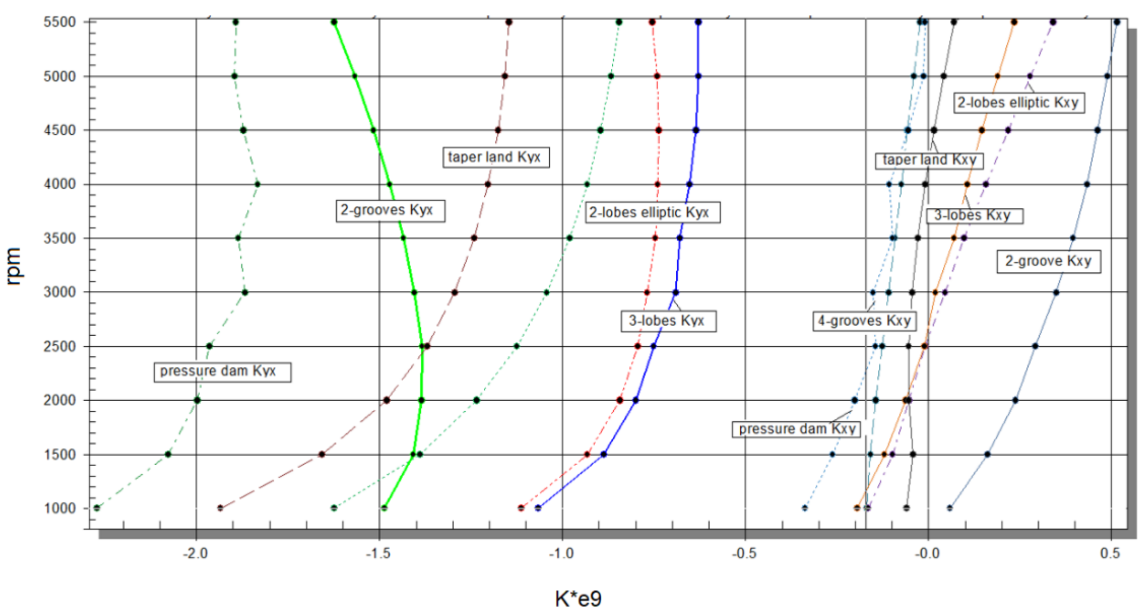

Pic. 9. Critical speeds map

The 4-grooves bearing that has 4 operating segments located at 45 degrees from the vertical axis shows the highest stability limit. This bearing stiffness is high enough to use it for the heavy high speed rotor. The other effects of the increased bearing stiffness are a critical speed increase and the operating speed margin reduction. Besides this the higher stiffness results in smaller damping efficiency.

The plot left part reflects performance of the four bearings with lower vertical stiffness. In this group the elliptic bearing shows a better stability marginand the best critical speed margin.

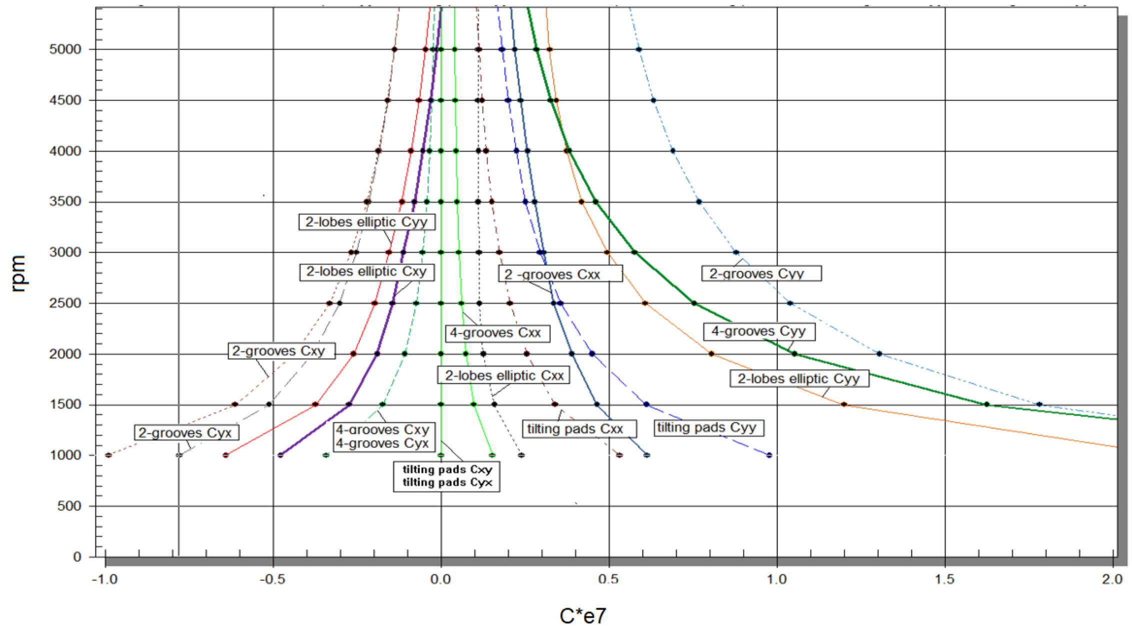
As mentioned above the cross-influence stiffness coefficients mostly determine the stability border position. Picture 10 shows these coefficients calculated for different bearings in the generator model.

A comparison of pictures 9 and 10 shows that the bearings with negative Kxy values at positive rotor rotation have the best stability limits.




Pic. 10. Cross-influence stiffness coefficients

The damping coefficients (pic. 11) are important parameters that finally determine the rotor vibration level. The analysis shows that the vertical direction damping coefficients Cyy are a few times higher than the horizontal direction Cxx ones. The cross-influence coefficients Cxy and Cyx usually have similar negative values which show increases of the oil film circulation forces and thus a reduction of the system stability border.

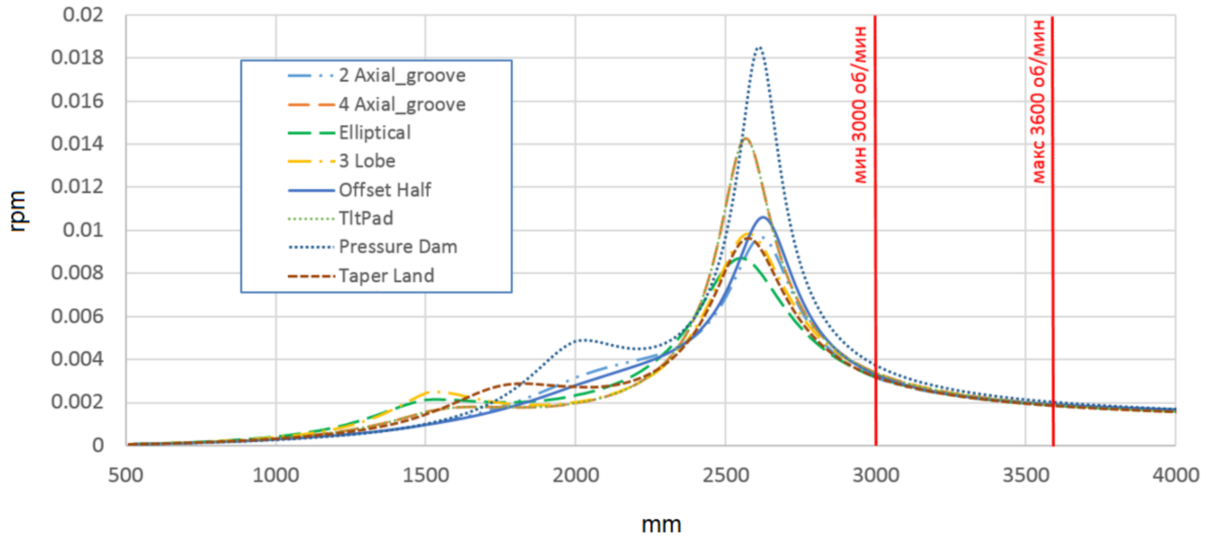



Pic. 11. Direct and cross-influence damping coefficients versus

It is worth mentioning that the 4 axial oil supply grooves bearing has 2…3 times smaller Cxy and Cyx values that the bearing with 2 grooves. These small values are the sources of the high stability border.. The tilting pad bearing has near to zero cross-influence damping coefficients.

Picture 12 shows the generator magnitude-frequency performance calculated for different bearing types. The elliptic bearing has the smallest resonance magnitude. Also the results show a good correspondence with the qualitative resonance assessment in the critical speeds map.

One more interesting result is the magnitude-frequency prerformance similarity of the 4 axial grooves and the 4 tilting pads bearings.




Pic. 12. Magnitude-frequency performance in the Y axis direction in the unbalance loction section.

Table 3 summarizes dynamic performance of the rotor on different bearing types. The results are obtained by the DYNAMICS R4 code and may be used for the structure optimization from the rotor dynamics point of view.

Table 3

| Bearing type | 2-axial groove | 4-axial groove | 2-lobes elliptic | 3-lobes elliptic | taper land | pressure dam | tilting pads |
|---|---|---|---|---|---|---|---|
| Stability border, rpm | 3872 | 9810 | 5060 | 4917 | 5622 | 4477 | - |
| Resonance frequency, rpm | 2590 | 2571 | 2552 | 2552 | 2617 | 2617 | 2495 |
| Support load at resonance, N | 1250 | 1876 | 1017 | 1012 | 1428 | 2491 | 1868 |
| Support load at operating speed, N | 400 | 445 | 400 | 472 | 432 | 500 | 500 |
| Power losses, KWt | 1.7e4 | 1.6e4 | 1.6e4 | 1.7e4 | 1.6e4 | 1.9e4 | 1.1e5 |
| Oil flow through faces, l/min | 12.17 | 8.99 | 30.15 | 11.19 | 12.2 | 14.45 | 10.01 |
| Necessary margin SMr | 15.2 | 15.7 | 14.5 | 15 | 15.1 | 16 | 15.7 |
| Actual margin SMA | 12.9 | 14.4 | 15.1 | 14 | 14 | 13 | 14.4 |

The analysis shows that all bearings have good damping and better stability margin than the 2-axial groove bearing. Other parameters are similar. Nevertheless, the only bearing that meets the API resonance tuning standard is the 2-lobes elliptic. Besides this bearing has a good stability margin. These two factors show that this bearing type provides the best rotordynamics performance for the rotor system. One more advantage of this type of bearing is its easy manufacturing. May be the only shortage of this configuration is its larger oil flow through faces than the other types.

## Conclusion

The paper describes the algorithm for selection of the sleeve bearing type at the rotor design stage. The selection is based on the stability border position, the stiffness and damping bearing performance, the distance resonance regimes from operation speeds and some other parameters. The stability is evaluated by the damping distribution along the operation regimes. The resonances location and their influence upon the generator workability are evaluated according to the API standards requirements.

The described algorithms are involved in the DYNAMICS R4 code and may be used for all types of machines with rotors supported by sleeve bearings.

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