

**CONSIDERATION OF CONTACT INTERACTION WHEN MODELLING STIFFNESS**  
**CHARACTERISTICS OF ROLL BEARINGS**

Degtiarev S.A., Kutakov M.N., Leontiev M.K., Popov V.V., Romashin Y.S.

Degtiarev Sergey A.; Engineering & consulting centre for dynamic problems in rotating machinery Alfa-Tranzit., Co.Ltd,1, Leningradskaya st., Khimky, Moscow region, 141400, Russia; Development team leader, [degs@alfatran.com](mailto:degs@alfatran.com)

Kutakov Maxim N.; Moscow Aviation Institute (national research university), MAI, 4, Volokolamskoe shosse, Moscow, A-80, GSP-3, 125993, Russia; student of 203 department, e-mail: [maxim.kutakov@alfatran.com](mailto:maxim.kutakov@alfatran.com)

Leontiev Mikhail K.; Moscow Aviation Institute (national research university), MAI, 4, Volokolamskoe shosse, Moscow, A-80, GSP-3, 125993, Russia; professor of 203 department, Doctor of Science (Technical), e-mail: [lemk@alfatran.com](mailto:lemk@alfatran.com)

Popov Valerii V., Bauman Moscow State technical university, 5, Baumanskaya st. 2<sup>nd</sup>, Moscow; lecturer assistant of applied mechanics department, e-mail: [vyvpopov.bmstu@gmail.com](mailto:vyvpopov.bmstu@gmail.com)

Romashin Yuri S.; Moscow Aviation Institute (national research university), MAI, 4, Volokolamskoe shosse, Moscow, A-80, GSP-3, 125993, Russia; student of 203 department, e-mail: [romashin@alfatran.com](mailto:romashin@alfatran.com)

**Abstract**

*When solving rotor dynamic tasks of rotating machines including aviation gas turbine engines, calculation accuracy of the system elements is highly important. One of the most widespread elements of the supports is a roll bearing. Its simulation is determined by necessity to take into account contact interaction between rolling elements and rings. Usually this interaction is described using analytical or empirical equations. This paper presents development of dependence between outer load and displacement for the tasks of linear contact of a cylinder half and a flat surface using finite element method. The obtained dependence is compared with an empirically obtained Palmgren equation for the same task.*

**Key words:** *roll bearing, stiffness, contact interaction*

## Introduction

When solving rotor dynamic tasks of rotating machines including aviation gas turbine engines, calculation accuracy of the system elements is highly important. General stiffness of the gas turbine engine support includes stiffness of bearings – ball or roll ones. Their stiffness depends on many factors – geometry, number of rolling elements, radial and axial loads, material characteristics, temperature loading, etc.

In the first place specific character of simulation of stiffness characteristics of roll bearings is related to contact phenomena between rolling elements and rings. Interaction between rolling elements is considered as inner contact between two cylinders along moving line. Analytical solution of this task obtained on the basis of contact theory of Hertz was published in the Lunberg-Shevall paper [1]. However, the obtained dependence between displacement and outer force is inconvenient to use in practical calculations, because it is impossible to express reverse relationship (force from displacement) in explicit form. The Harris paper [2], one of the based ones to analyze roll bearings, presents such dependence using empirical Palmgren equation. This dependence may be deduced more grounded by using numerical methodologies based on the finite element method (FEM). The present work shows how dependence between outer force and

displacement was deduced in the task of contact between a cylinder half and a flat surface using FEM.

## General theory

According to analytical solution of the contact task about interaction between two cylinders published in the Lunberg-Shevall paper [1], displacement in the contact point is the following

$$\delta = \frac{2Q(1-\nu^2)}{\pi El} \ln \left[ \frac{\pi El^2}{Q(1-\nu^2)(1 \pm \gamma)} \right] \quad (1)$$

where  $Q$  – outer acting load,  $E$  – Young modulus,  $\nu$  – Poisson ratio,  $l$  – cylinders length,  $\gamma$  – ratio of cylinders diameters (sign depends on mutual position of the cylinders between each other).

Empirical dependences are used in practice.

One of widespread equations is Palmgren's:

$$\delta = 3.84 \cdot 10^{-5} \frac{Q^{0.9}}{l^{0.8}} \quad (2)$$

Expressing  $Q$  through  $\delta$ , we obtain equation used in Harris paper [2]

$$Q = 8.06 \cdot 10^4 l^{\frac{8}{9}} \delta^{\frac{10}{9}} \quad (3)$$

We write (3) like Hooke law

$$Q = \left( 8.06 \cdot 10^4 l^{\frac{8}{9}} \delta^{\frac{1}{9}} \right) \delta = K \delta, \quad (4)$$

where  $K = 8.06 \cdot 10^4 l^{\frac{8}{9}} \delta^{\frac{1}{9}}$  – coefficient of contact stiffness.

### Problem statement and description of finite-element method

To obtain relationship between outer load and displacement, the task of contact between the cylinder half and the flat deformable surface (the cylinder with infinite radius) was solved at different lengths of a contact line (Figure 1). Length varied from 0.5 to 50 mm.

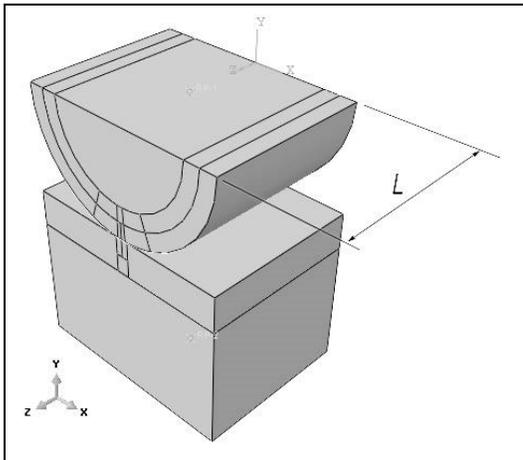


Figure 1 Statement of task of contact between cylinder and plane.  $L$  – length of contact line (variable value)

To simulate, hexahedral linear 8-nodes finite elements were used. Figure 2 shows a finite-element mesh. There is the mesh densening in the contact place. The nodes number in the model is 62096. Figure 3 shows places of loading and fixing. The model is fixed at the surface at all freedom degrees. Load is applied at the surface at direction that is opposite to Y-axis, loading value is 2000 N. Material is isotropic, Young modulus is  $2 \cdot 10^5$  MPa, Poisson

ratio is 0.3. Contact task is solved using Lagrange method.

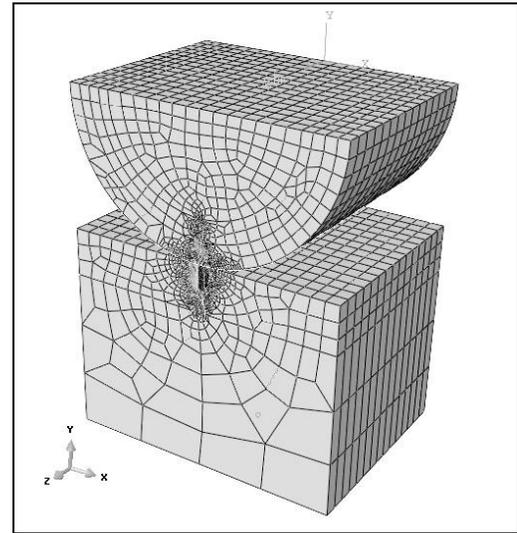


Figure 2 Finite-element mesh of model

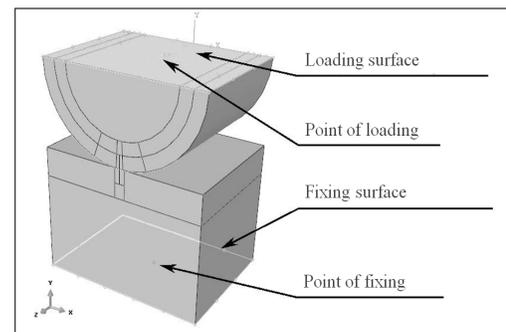


Figure 3 Places of loading and fixing

### Calculation results

Data obtained as calculation results are presented as dependence of vertical displacement from length of contact line (Figure 4). The plot shows solutions obtained using Palmgren's and Lundberg-Shevell equations.

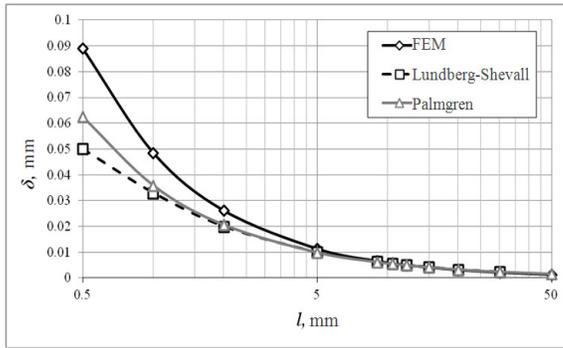


Figure 4 Dependence of vertical displacement from cylinder length using different calculation methods: solid curve with squares– calculation using FEM, dotted curve with triangles– using Palmgren’s equation, dashed line with circles – using Lundberg-Shevall equation

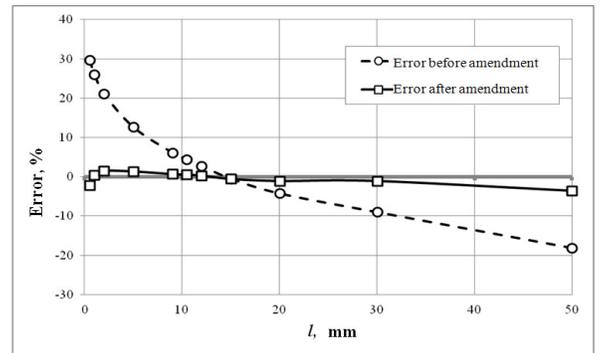


Figure 5 Relative difference between solutions obtained using Palmgren’s formula (dashed line with circles) and using equation (5) (solid curve with squares) comparing with FEM solution

However, the diagram of relative difference between solutions obtained by FEM and Palmgren (Figure 5) shows that Palmgren formula is correct with error of 5% for lengths of contact lines from 10 to 20 mm.

After approximation of results obtained using FEM methods, the following was found out

$$Q = 57865l^{1.011}\delta^{\frac{10}{9}}$$

or

$$Q = \left(57865l^{1.011}\delta^{\frac{1}{9}}\right)\delta = K\delta \quad (5)$$

Equation (5) gives no more than 4% divergence with calculation using FEM (Figure 5).

Figure 6 gives contact stiffnesses obtained using Palmgen’s equation and equation (5).

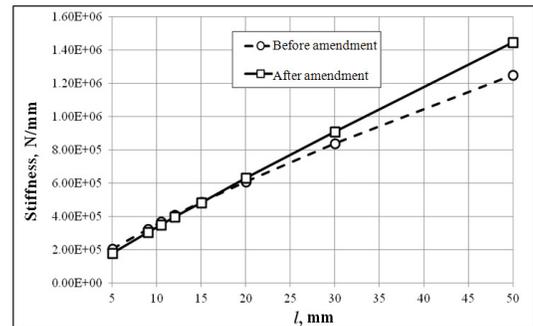


Figure 6 Values of contact stiffnesses

## Conclusion

This paper shows results of task about contact interaction between the cylinder half and the plane at different lengths of contact line (from 0.5 to 50 mm) using FEM. The obtained results are compared with the empirical Palmgren equation. It is shown that the Palmgren equation gives the results differing from the FEM solution in up to 30% for the

contact lengths from 0,5 to 50 mm. By approximation of the obtained results, the new equation was deduced. It gives error of no more than 3% at all range from 0,5 to 50 mm of investigated lengths of contact lines. The obtained results may be used in engineering offices, designing aviation gas turbine engines and particularly when solving rotordynamic tasks.

## **Literature**

1. Lundberg, G. and Sjövall, H. *Stress and Deformation in Elastic Contacts*, Pub. 4, Institute of Theory of Elasticity and Strength of Materials, Chalmers Inst. Tech., Gothenburg, 1958. 47p.
2. Harris T.A. *Rolling bearing analysis*. New York, John Wiley and Sons, 1984, ch. 6-7, pp. 101-159 (565 p.).